Electromagnetic Efficiency of Block Design in Superconducting Dipoles

L. Rossi and E. Todesco

***Abstract*—Semi-analytical scaling laws have been derived for the electromagnetic design of superconducting dipoles based on sector coils approximation. Here we discuss the different features of block coils designs used in the past, focusing on the field quality issues, and on the efficiency of the electromagnetic design (i.e. short sample field versus the quantity of superconductor) with respect to the standard cos theta design.**

***Index Terms*—Current density, stress, superconducting magnets.**

I. Introduction

MAGNET design can become as passionate as a soccer match when one has to select the "optimal" coil layout. It is amazing to see how many discussions can be triggered by the way of placing a cable around a beam tube. Since the construction of the first superconducting magnets for high energy particle accelerators, the community has witnessed strong debates between the fans of the so-called $\cos\theta$ design and the supporters of the block design (see Figs. 1 and 2). In the first case, the coil has an arch shape and it is made of sectors of Rutherford cables, usually keystoned, separated by wedges aiming at approximating a $\cos\theta$ current density distribution ([1]–[4], see Fig. 1). In the second case, the cable and the blocks are rectangular, and they are arranged along vertical and horizontal guidelines ([5]–[9], see Fig. 2). The main open issues are the possibility of reaching a sufficiently uniform field quality, the efficiency of the design, and the related mechanical structure. Even though both designs have been successfully used to build models, superconducting magnets used in the four existing high energy accelerators all rely on a $\cos\theta$ design [10]–[13].

In [14] we developed a semi-analytical approach to the electromagnetic design of superconducting dipoles based on the $\cos\theta$ layouts. This work provided an educated guess of the achievable field in a dipole as a function of the main parameters without the need to go through a detailed design. Moreover, it gave a benchmark to judge the efficiency of any design. Here, we apply these tools to study the designs based on block coils that have been proposed in the literature [5]–[9], or that have been built in short models.

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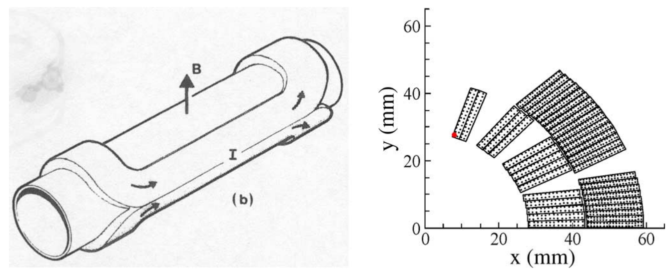

Fig. 1. 3D sketch of a $\cos\theta$ coil from [1] (left), and cross-section (right, one quarter) of the LHC dipole [13].

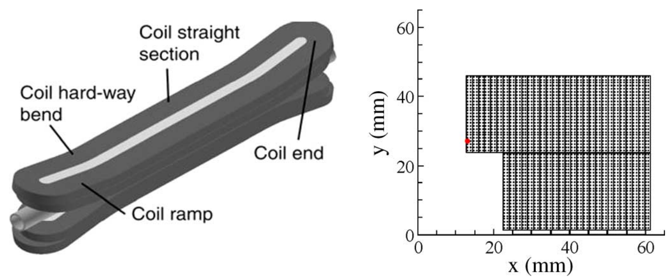

Fig. 2. 3D sketch of a block coil from [9] (left), and cross-section (right, one quarter) of the LBNL-HD2 model.

In Section II we present the different layouts. In Section III we study the constraints given by field quality. The efficiency of the design in terms of maximum field achieved for a given quantity of superconductor is discussed in Section IV.

II. Coil Layouts

A. Sector Coil Design $(cos\theta)$

The coil layout which has been used in almost all accelerator main magnets is based on sector coils made up of Rutherford cables. Cables are packed in blocks, separated by wedges to form arches (see Fig. 1). One or more layers can be used to get the required field (see Figs. 1 and 3). We remind the reader that in a dipole based on sector coils the central field is proportional to the current density j and to the width w of the coil according to the equation.

$$B = j \cdot \gamma = \gamma_0 j w, \tag{1}$$

where the field is expressed in T, the current density in $\mathrm{A/m^2}$, the coil width in m and γ_0 is a constant depending on the coil geometry. For instance, for a 60° sector coil without wedges one has $\gamma_0 = 6.93\ 10^{-7}$ Tm/A [14].

Rutherford cables can be either slightly keystoned or rectangular. In the first case the coil can better follow the arc shape

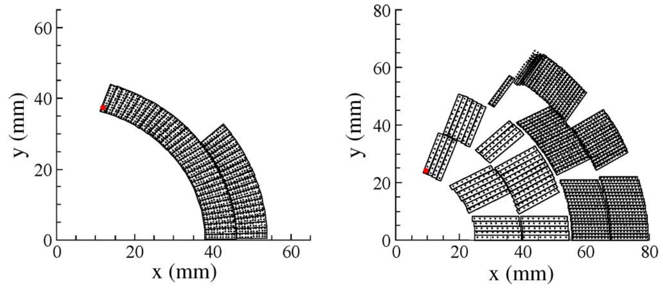

Fig. 3. Coil layout of the Tevatron [10] dipole (left) and of the LBL-D20 model [15] dipole (right), one quarter shown.

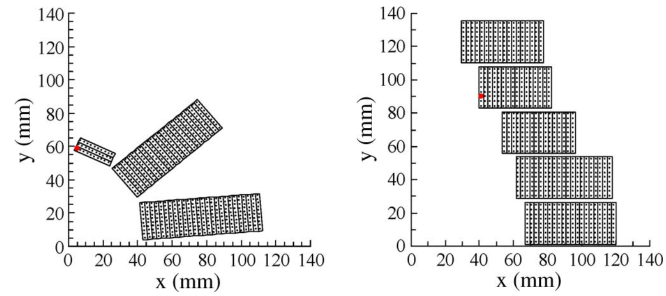

Fig. 5. Proposed coil layout of the NED [16], motor winding (left) and ellipse-type (right), one quarter shown.

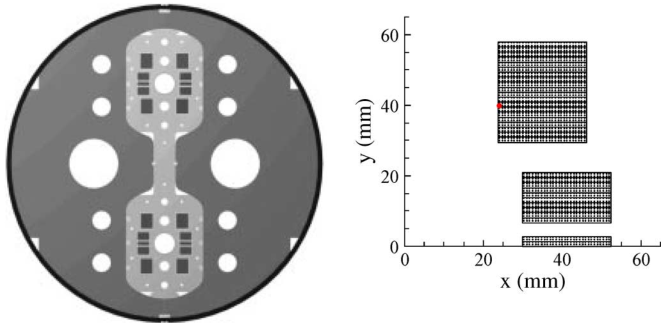

Fig. 4. Second version of the coil layout of the common coil model HFDC, cross-section (left), and coil layout in one quarter of the aperture (right) [7], [8].

even in the case of very wide blocks. A completely keystoned cable was used in the Tevatron dipoles (see Fig. 3, left), allowing to have no wedge. This design has only a limited capability of minimizing field harmonics (see next section). For this reason, all the layouts used in the more recent magnets have wedges. Even with a non keystoned cable one can build a $\cos\theta$-like coil (see for instance D20, right Fig. 3) by an appropriate use of wedges and by avoiding building blocks that are too large with respect to the needed curvature radius of the layer. The main feature and advantage of the $\cos\theta$ layout is that the coil is self supporting (being an arch) w.r.t. azimuthal stress, i.e. there is no need of an internal structure and therefore all aperture is available for the beam.

B. Common Coil Design

A common coil layout [5] produces a vertical magnetic field in two apertures, vertically aligned, with opposite directions (see Fig. 4, right). It is made up with Rutherford cables which are arranged in rectangular blocks, with the wide side of the cable parallel to the midplane. Let us consider the upper right quadrant of the upper aperture: the return coil will not be in the upper left quadrant as in the $\cos\theta$, but in the lower right part of the lower aperture (see Fig. 4). The curvature radius at the coil ends corresponds to the distance between the apertures: this means that, with respect to the $\cos\theta$ layout, the required bending is much less. Indeed, this layout has been proposed for winding reacted $\mathrm{Nb_3Sn}$ cable ("react and wind" technology), which is extremely brittle and cannot tolerate the strain induced by small curvature radii. The coil is not self supporting and an internal structure is needed (see Fig. 4).

C. Block Design

In the block design the coil is made up of rectangular blocks of Rutheford cable, whose wide sides are vertical, i.e. perpendicular to the midplane (see Fig. 2, left). This is a single aperture magnet, with a vertical main field. Coils are built as racetrack, with the ends bent upward or downward to leave space to the beam aperture (see Fig. 2, right). This design has been successfully implemented in the $\mathrm{Nb_3Sn}$ model HD2 [9] which presently has the highest reported magnetic field in a superconducting accelerator magnet. The curvature radius of the cable in the ends is similar to the $\cos\theta$ layout, and therefore in the case of $\mathrm{Nb_3Sn}$ a wind and react technique has to be applied. As in the common coil case, an internal structure is needed.

D. Other Proposals

The Next European Dipole (NED) project aims at building a 15 T dipole with a large bore of 44 mm aperture radius. Several coil layouts have been developed [16]; among them we consider the motor winding (Fig. 5, left) and the ellipse-type (Fig. 5, right). The motor winding has the main feature of being able to avoid the accumulation of stress on the midplane, whereas the ellipse type has conductors placed such as in HD2, but aiming at reproducing an intersecting ellipse. The designs are for the moment just on paper and drafts, and solutions for the coil heads should be worked out.

III. Field Quality

A. Definitions and General Remarks

The field homogeneity for an accelerator magnet must be of the order of 10^{-4} over a relevant fraction of the aperture (typically 2/3) which can be occupied by the beam. Usually one defines field harmonics according to the expression

$$B(z) = \sum_{n=1}^{\infty}(B_n + iA_n)\left(\frac{z}{R}\right)^{n-1} = 10^{-4}B_1\sum_{n=1}^{\infty}(b_n + ia_n)\left(\frac{z}{R}\right)^{n-1} \tag{2}$$

where the harmonics are usually normalized to the main component B_1. The reference radius R is usually chosen as 2/3 of the aperture radius r. The field quality constraint for accelerator magnets is to have all harmonics up to order 11 smaller than 1.

In the case of a distribution of current with uniform density j, the harmonics are given by

$$B_n = -2\frac{\mu_0 j}{2\pi}R^{n-1}\int_{r}^{r+w}\int_{\theta_0}^{\theta_1}\rho^{-n}\cos(n\theta)\rho d\theta d\rho. \quad (3)$$

As is well known, coils with a dipole symmetry only have odd non zero harmonics $b_3, b_5, b_7, \ldots$, and a 60° sector coil sets to zero b_3, leaving a residual b_5 which is a function of the aperture and the coil width

$$b_5 \equiv \frac{1}{B_1}B_5 \propto \frac{1}{w}\left[\frac{1}{(r+w)^3} - \frac{1}{r^3}\right]. \quad (4)$$

This equation simply shows that $b_5 \to 0$ as $w \to \infty$, i.e. field quality constraints are easier to satisfy for cases in which the ratio between the coil width w and the aperture radius r is large (>1, as for instance in HD2). In these cases, the outer part of the coil gives a relevant contribution to B_1, and negligible contributions to the higher orders B_n [see (3)], and therefore the normalized multipoles b_n are small. Coils with $w/r \gg 1$ "naturally" give rise to good field quality. On the other hand, thin coils such as the RHIC dipole are more difficult to optimize. This also reflects on the ratio between the central field and the peak field in the coil, as it will be discussed in Section IV.

B. Sector Coil Design ($cos\theta$)

Having three sectors delimited by the angles (θ_5, θ_4), (θ_3, θ_2), and $(\theta_1, 0)$, (we assume a single layer, without opening on the midplane, see [17] for the analysis of an open midplane coil) one has 5 free parameters and therefore integrating (3) one can set to zero the first five 'allowed' multipoles B_3 to B_{11} by solving the system

$$\begin{aligned}
\sin(3\theta_5)-\sin(3\theta_4)+\sin(3\theta_3)-\sin(3\theta_2)+\sin(3\theta_1)&=0\\
\sin(5\theta_5)-\sin(5\theta_4)+\sin(5\theta_3)-\sin(5\theta_2)+\sin(5\theta_1)&=0\\
\sin(7\theta_5)-\sin(7\theta_4)+\sin(7\theta_3)-\sin(7\theta_2)+\sin(7\theta_1)&=0\\
\sin(9\theta_5)-\sin(9\theta_4)+\sin(9\theta_3)-\sin(9\theta_2)+\sin(9\theta_1)&=0\\
\sin(11\theta_5)-\sin(11\theta_4)+\sin(11\theta_3)-\sin(11\theta_2)+\sin(11\theta_1)&=0
\end{aligned} \quad (5)$$

The physical meaning of this set of equations is to approximate the $\cos\theta$ function (which provides a perfectly homogeneous dipolar field) with a sum of square waves (the sectors). In general an optimization of multipoles up to B_{11} is enough to guarantee a sufficiently homogenous magnetic field. Some free parameters are used to compensate the fact that the block angular widths are not continuous variables, due to the finite thickness of the Rutheford cable. On the other hand, additional parameters for the fine tuning of field quality are provided by the possibility of slightly tilting the blocks (up to ±10°). All the coils built in the past have at least three blocks, and therefore the optimization problem is under-constrained. The only exception is the Tevatron coil, where having two layers without wedges one can only set to zero B_3 and B_5. This degree of homogeneity was considered to be sufficient in the 80s, but the beam dynamics requirements have become more and more stringent since then.

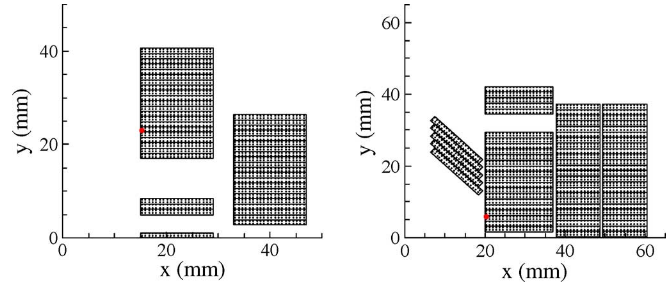

Fig. 6. Coil layout of the first version of the common coil HFDC (right), and of a VLHC dipole design using correcting coils (left).

The sector coil design has been considered for a long time as the most appropriate to get a good field quality. Indeed, sector coils have no specific properties that make them more effective to produce pure dipolar fields. The only remarkable feature is that the field quality equation for one layer (5) is independent of the width of the coil, thus making the optimization equations simpler. Indeed, most of the coils rely on more than one layer, and usually each layer is not giving a pure field, but layers are compensating each other. Therefore, this special feature of the $\cos\theta$ is not used in the Tevatron, HERA and LHC dipole layouts.

C. Common Coil Design

An aperture between two "walls" of finite thickness w and infinite height $h = \infty$ provides a pure dipolar field. When a finite h is considered, high order components appear. For instance, if we consider a coil width equal to the aperture radius $w = r$, the coil height has to be at least 3 times the aperture $h > 3r$ to have all multipoles smaller than one unit, with the exception of a residual b_3 of ~50 units. A one-layer coil without wedges should be very tall to provide an accelerator-like field quality, but this is a waste of cable and makes the magnet very large. The only solution is to insert wedges. Since the top of the coil is too distant from the beam, high order multipoles can only be optimized by inserting wedges close to the midplane. This explains why the HFDC coils shown in Figs. 4 and 6 (left) have large wedges very close to the midplane. Thanks to these large wedges, all multipoles are smaller than 1 unit. Indeed, these midplane wedges make the design less effective, as we will show in Section IV. Another possibility is putting correcting coils [6], as shown in Fig. 6, right, much closer to the vertical midplane. This breaks the initial simplicity of the design, but gives the possibility to have less sparse coils in the midplane, providing a more effective design. The feasibility of this design with an appropriate mechanical structure should be analyzed.

D. Block Design

The HD2 block design (see Fig. 2) achieves an accelerator-like field quality with multipoles of the order of 1 unit. This layout looks remarkably simpler than the common coils shown in Figs. 4 and 6: it has two layers but no wedges at all. A crucial part of the design is that the upper layer is much closer to the vertical axis than the midplane layer. This avoids the need of wedges in the midplane: however, it implies a vertical support

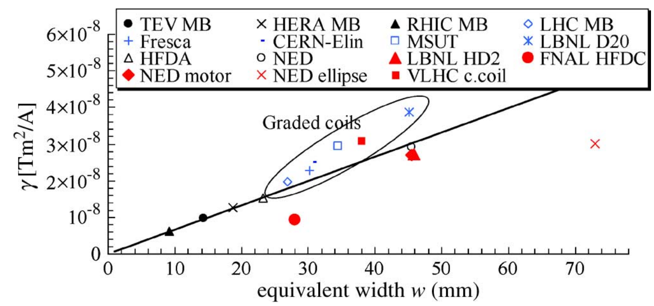

Fig. 7. Ratio central field/current density versus equivalent coil width for 5 examples of non $\cos\theta$ design (red markers) for some $\cos\theta$ layouts (small markers) and linear relation for a [0°, 48°], [60°, 72°] sector coil (solid line).

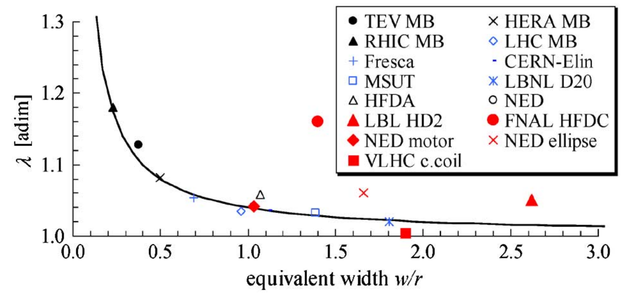

Fig. 8. Ratio peak field/central field versus equivalent coil width for 4 examples of block design, (red large markers) for some $\cos\theta$ layouts (small markers) and fit for a sector coil (solid line).

for the upper coil on the inner side, thus reducing the mechanical aperture. The reduction depends on the detailed design. The second favorable feature is that HD2 has a larger coil width than both HFDC and VLHC (for a similar aperture), and therefore the higher order harmonics are closer to zero (see Section III-A).

IV. Short Sample Field

The short sample field is the maximum performance of the magnet when the current density is on the critical surface for the corresponding peak field. It is given by a combination of different features: how much field is given by a certain current density, and the ratio between the peak field in the coil and the central field. In Fig. 7 we give the ratio γ between the field in the bore and the current density in the coil. For a sector coil this parameter is proportional to the coil width w

$$\gamma = \gamma_0 w \tag{6}$$

with $\gamma_0 = 6.625 \times 10^{-7}\ \mathrm{Tm/A}$ for a two-sector coil canceling b_3 and b_5 as the [0°, 48°], [60°, 72°] case. The agreement of this simple approximation with the $\cos\theta$ dipole layouts is remarkable [14]. Here we also give the same factor evaluated for the block designs. In this case, w is defined as the width of a 60° sector coil having the same area A of the block coil

$$w_{eq} = \left(\sqrt{1 + \frac{3A}{2\pi r^2}} - 1\right) r. \tag{7}$$

The HFDA common coil and the NED-ellipse give 35–50% less field for the same current density j and coil quantity A (see Fig. 7); for the HD2 and motor winding one only has a 10% loss. The VLHC common coil has a 50% grading, and gives ~20% higher field for the same j and coil quantity A.

The ratio λ between peak field and central field is a function of the coil shape. In [14] we proposed an empirical fit

$$\lambda = 1 + a\frac{w_{eq}}{r} \tag{8}$$

for the $\cos\theta$ layouts, with $a = 0.04$ (see Fig. 8). This means that in the case of a coil width equal to the aperture radius $w = r$, one has $\lambda = 1.04$, i.e. the peak field in the coil is 4% larger than the central field. For larger coils, the ratio tends to one. The common coil without correcting coil HFDA has a much less favorable ratio (13% more), whereas the motor winding, the

TABLE I
Difference Between γ, λ and Short Sample Field of the Four Block Layout w.r.t. the $\cos\theta$ Benchmark

	Difference with respect to cosθ benchmarks (%)		
Magnet	γ	λ	B_{ss}
HD2	-12	4	-5
FNAL HFDC01	-49	13	-22
VLHC comm. Coil	19	-2	5
NED motor winding	-12	1	-3
NED ellipse	-38	3	-9

ellipse, and HD2 have a rather similar ratio (up to 3.5% more) (see Fig. 8). The common coil with correcting coil VLHC is very well optimized and its peak field/central field ratio is very close to one.

Both the lower efficiency of the coil in producing central field for a given current density and quantity of cable (ratio γ) and the lower efficiency of the coil in producing central field for a given peak field (ratio λ) reduce the short sample field. Both γ and λ also depend on the shape of the critical surface, and it is different for Nb-Ti or $\mathrm{Nb_3Sn}$. We computed the short sample loss for the analyzed layouts, comparing to a $\cos\theta$ coil made with the same quantity of cable. One obtains a loss of about 20% for the common coil without correcting coils, 8.5% for the ellipse, and of 3–5% for HD2 and for the motor winding. The VLHC common coil, with correcting coils and strong grading, has a short sample field which is 5% larger than the benchmark. For a generic $\cos\theta$ design, the agreement with the $\cos\theta$ benchmark is in general within ±2% (Table I).

V. Conclusions

All designs can give pure homogeneous fields, if they have a sufficient number of blocks. The $\cos\theta$ lay-out has not any special features that make it more apt to give pure fields with respect to other coil layouts. Field quality in coils with large widths with respect to the aperture is easier to optimize.

The comparison of the short sample performance points out that both the HD2 layout and the motor winding layout give the same short sample field (within 5%) as a $\cos\theta$ if we fix the quantity of cable. On the other hand, the ellipse type and the common coil have a non-negligible loss in the efficiency, as they provide 10–20% less short sample field for the same coil area. Using correcting coils, a common coil can perform as a $\cos\theta$, but the mechanical structure needed for this design should be carefully studied. Other aspects such as stress due to electromagnetic forces, mechanical structure, and manufacturability should also be taken into account, and are not treated in this paper.

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